

PROPERTY OF
GETON UNIVERSITY LIBRARY

RECEIVED JUL 20 1939

The
PSYCHOLOGICAL
RECORD

SW
JULY, 1939
Vol. III No. 10

THE EFFECT OF PARTIALLING ON TWO FACTOR
METHODS: THURSTONE AND KELLEY
(MODIFIED)

W. J. E. CRISSY



THE PRINCPIA PRESS, INC.
BLOOMINGTON, INDIANA

Price of this number, 15 cents

THE EFFECT OF PARTIALLING ON TWO FACTOR METHODS: THURSTONE AND KELLEY (MODIFIED)*

By W. J. E. CRISSEY

University of North Carolina

The purpose of this paper is threefold: (1) to present a worked example of factorial analysis to demonstrate the invariant nature of factor loadings¹, (2) to show the effect upon factor loadings of partialling out a variable, (3) to offer an example for comparing the Kelley and Thurstone solutions in the case of unpartialled and partialled variables. The data used in the example were developed in connection with a study of the physical indices of pubescence, and the example itself bears on statistical problems of general interest.

Factorial analyses of five "unpartialled" indices of pubescence and four "age-partialled" indices of pubescence have been made by the two methods mentioned above.² The five variables have known factor loadings with respect to "meaningful" factors found in the solution of a ten variable problem by the Thurstone technique (reported elsewhere**). While these reported factor loadings are resultant from a Thurstone solution it has been elsewhere demonstrated³ that the Kelley-Hotelling⁴ components, when

* Recommended for publication by Dr. J. P. Guilford, May 15, 1939.

¹ Thurstone, L. L., "Current Misuse of the Factorial Method," *Psychometrika*, II, 2, 1937, pp. 73-76.

² The procedure for the Kelley analysis is found in Kelley, T. L., *Essential Traits of Mental Life*, Harvard University Press, 1935. The Thurstone analysis is explained in Thurstone, L. L., *The Vectors of Mind*, University of Chicago Press, 1935. Rotation of axes was done graphically rather than by the computational method explained in Thurstone's treatise.

^{**} "Unitary Traits of Pubescence," A.A.A.S., December, 1938, Richmond.

³ McCloy, C. H., Methany, E., and Knott, V., "A Comparison of the Thurstone Method of Multiple Factors with the Hotelling Method of Principal Components," *Psychometrika*, III, 2, 1938, pp. 61-67.

⁴ Kelley, *op. cit.*, p. 1, states "The procedure followed is new, but the outcome is identical with that given by Hotelling's method of analysis."

rotated into meaningful configurations, yield approximately the same results as the Thurstone solution.

The subjects in the present study are 166 white public school girls.

The variables treated are: 1. *chronological age*, 2. *standing height*, 3. *amount of public hair* (a four point rating scale estimate of quantity of public hair where 0 = absence, 1 = presence but unpigmented, 2 = presence and slightly pigmented, 3 = abundant, pigmented, and fully kinked), 4. *amount of axillary hair* (a four point rating scale estimate of quantity of axillary hair by the same method as used in 3), 5. *eruption of second molar teeth* (a three point rating scale estimate in which 0 = not erupted, 1 = erupting, 2 = erupted).

Before presenting the illustrative problem it might be well to summarize the statements made by Stephenson⁵, Kelley⁶, and others with regard to this procedure. To quote Stephenson⁷, "This partialling, whilst a fit and proper device for partialling out variables like *age*, or *height*, or *weight*, cannot be used with impunity for partialling out variables of the kind supplied by psychological tests, unless the tests are themselves unitary measures of what they purport to measure." Kelley adds to this argument that any objectively determinable rubric which has known correlates should be kept in its own "pure" state by partialling it out of more complex variables.

It seems to the writer, however, that there are many rubrics which appear superficially to be unitary but which when analyzed in specific settings may show complex weightings on several "meaningful" factors. For example, it will be shown that age carries a significant factor loading on "Sexual Development" but no loading on "Calcium-skeletal Development." This does not imply that age is unrelated to such obvious correlates of skeletal development as height, weight, carpal ossification, etc., it means that in a medically recognized component of pubescence, "Calcium-skeletal Develop-

⁵ Stephenson, W., "A Note on Factors and the Partial Correlation Procedure," *British Journal of Psychology* (general section) XXV, 1935, pp. 399-401.

⁶ Kelley, T. L., *op. cit.*, IV.

⁷ Stephenson, W., *op. cit.*, p. 399.

ment," it carries no loading.⁸ Probably in a study dealing with subjects of *all* ages, age would have a loading on this factor.

One of the most fruitful contributions of factorial procedure has been the establishment of basic relationships among available metrics in various fields, especially in psychometry. To partial out any variable which looks as though it may have differential discriminative value in the finding of these basic relationships seems to the writer to be setting up, *a priori*, serious limitations on the outcome of the factorization. If a variable exists which is wholly unitary, i.e., totally homogeneous in construct, it should show itself in the factorization as a specific factor.

Table 1 gives the original correlation matrix. Table 2 gives the same matrix with variable 1, *chronological age*, partialled out of the remaining variables. In making the Kelley analysis of these data estimated reliability coefficients were used.⁹

TABLE 1
ORIGINAL CORRELATIONAL MATRIX

	2	3	4	5
1	.331	.480	.506	.148
2		.556	.547	.209
3			.761	.225
4				.169

TABLE 2
"AGE-PARTIALLED" CORRELATIONAL MATRIX

	3	4	5
2	.480	.502	.211
3		.684	.199
4			.110

⁸ The author is grateful to Dr. H. Ward Ferrill of the University of North Carolina Medical School Faculty for suggestions concerning the factor identification.

⁹ The values of the reliability coefficients as estimated:

1. 1.000 2. .980 3. .900 4. .850 5. .850

When partialled these become:

2. .978 3. .870 4. .798 5. .847

Table 3 gives the unrotated and rotated Thurstone factors. Table 4 gives the unrotated and rotated Kelley factors.¹⁰ Table 5 gives the factor loadings of the same variables in the ten variable problem previously referred to.

TABLE 3
THURSTONE SOLUTION

	Unrotated				Rotated			
	I	i*	II	ii*	I'	i'*	II'	ii''*
1	.603		—.155		.621		.010	
2	.673	.667	.203	.158	.595	.570	.378	.398
3	.852	.906	—.108	—.210	.842	.840	.120	.100
4	.840	.780	—.238	—.312	.878	.850	—.010	.000
5	.299	.288	.246	.290	.220	.170	.320	.390

*minuscule figures refer to "partialled" data factors.

With regard to the effect of partialling, if we inspect pairs of rotated axes in Table 3 we see a striking similarity between the unpartialled and partialled results. The results indicate that the Thurstone solution of these data yield similar loadings for a variable whether "age-partialled" or not, so that the method itself seems to "partial out" any overlapping among the variables. In contrast the same results are not obtained with the Kelley solution as inspection of pairs of axes in the "Rotated" section of Table 4 will reveal. With the exception of the heavily loaded factor, III' iii', little consistency is seen between the unpartialled and partialled variables.

With regard to the invariant nature of the factor loadings, if we compare columns B and C of Table 5 each separately with column A, we note a general correspondence of loadings although both sets of computed loadings have "overestimated" the communality for each of the variables. A similar comparison of the entries in columns a, b, c of the same table reveals about the same rough agreement with the new variables again "overestimating" the original communalities. It will also be noted that in the case of both factors (A and

¹⁰ The regression coefficients yielded by the Kelley solution were multiplied, each set by the standard deviation of the component involved, thus yielding saturation coefficients which are the entries in the "unrotated" columns of Table 4.

TABLE 4
KELLEY SOLUTION**

	I	i*	II	Unrotated					v*
				ii*	III	iii*	IV	iv*	
1	.612	.251		.706		.799	-.018	-.031	-.263
2	-.476	.422	.587	.763		.849	-.010	-.042	.028
3	-.108	-.298	-.311	.862		-.230	-.250	-.046	-.125
4	-.049	-.288	-.163	.833		.806	.238	.273	-.113
5	.206	.002	.130	.341		.349	.019	.045	-.240
									.794
Rotated									
	I'	i'*	II'	ii'*	III'	iii'*	IV'	iv'*	v'
1	.740		.210		.660		.000		-.060
2	.530		.490	.695	.700	.540	.090	.150	-.160
3	-.020		.200	.440	.830	.800	.520	-.090	.130
4	-.060		.170	-.010	.850	.800	.020	.400	.050
5	-.030		.610	.650	.200	.100	.100	.000	.570
									.620

*ibid.

**The Kelley analysis yields as many final components as there are initial variables.

TABLE 5

COMPARISON OF FACTOR LOADINGS WITH KNOWN LOADINGS

	A*	B**	C***	a*	b**	c***
1	.485	.621	.660	.000	.010	.210
2	.345	.595	.700	.270	.378	.490
3	.690	.842	.800	.170	.120	.200
4	.730	.878	.850	.000	-.010	.170
5	.000	.220	.200	.300	.320	.610

* A Loadings on "Sexual Development" from 10 variable problem.

** B Loadings found in column I' of Table 3

*** C Loadings found in column III' of Table 4.

* a Loadings found on "Calcium-skeletal Development" from 10 variable problem.

** b Loadings found in column II' of Table 3.

*** c Loadings found in column II' of Table 4.

a), the Thurstone solution gives what would appear to be a better fit to the original loadings.¹¹ Thurstone points out that his method of successive approximations to the communality is likely to result in errors of estimating the true communality, that are larger, the smaller the number of variables. In fact, except for illustrative purposes, a number of variables as small as five with as many factors as two known to be present would be unfeasible.

The other factors yielded by the Kelley solution (I'i', IV'iv', V'v') are perhaps to be interpreted as heavily loaded specific factors. These heavy loadings may be due to initial overestimation of the communalities by using the reliability coefficients in the diagonal cells of the original matrix.

If the results obtained in this admittedly limited problem are indicative of what we may expect from the use of the two methods it would seem that the Thurstone method is likely to be less affected by partialling than the Kelley method and it is also likely to yield weightings that are more invariant than the weightings yielded by the Kelley solution.

Addendum: Dr. John Flanagan points out that explicit men-

¹¹ Since, as Kelley, *op. cit.*, points out, there is no method of computing the standard error of these loadings, only subsequent developments in factor theory; or specific empirical information in the case of specific situations can really demonstrate what constitutes a "good" fit.

tion should be made that this Kelley treatment is really an example of a modification of the Kelley method (cf: footnote 10), that Kelley does not subscribe to such rotations of axes on the grounds that such a procedure prevents a unique solution.

Flanagan also calls attention to the fact that the "criterion weights" are resultant from a Thurstone solution and this "loads the dice" against the Kelley fit to these same data.

OC
R

Pri